EE 242  EXPERIMENT 4  PHASOR DIAGRAMS  

OBJECTIVE

To develop the ability to construct accurate phasor diagrams for understanding KVL in ac circuits.

EQUIPMENT

1 Agilent 33120A Function Generator
1 Agilent 34401A Multimeter
1 Agilent 54622A Oscilloscope
2 Decade Resistance Boxes
1 8H Inductor
1 Decade Capacitor
2 BNC-Banana
6 Banana-Banana

BRING A STRAIGHT-EDGE, SCALE, PROTRACTOR, AND 6” COMPASS TO THE LAB.

DISCUSSION

Figure 1(a) shows a simple ac circuit with series RL components. L and R_L are lumped parameters representing quantities that are distributed throughout the entire length of the coil of wire. The Kirchoff’s Voltage Law for the circuit may be described by drawing a phasor diagram to show both magnitude and phase angle of the voltages in the circuit. Figure 1(b) illustrates such phasor diagram. Note that circuit current is chosen to be the reference for the phasor diagram since current is common to all components in the circuit. In the lab, the magnitudes of V_s, V_L, and V_A are measurable, but not so for V_L and V_RL. This is where phasor diagrams may be useful.

![Figure 1. (a) A simple series RL ac circuit (b) Phasor Diagram for series RL ac circuit](image)

1 Original experiment, amended and revised 02/16/05, Taufik
To determine magnitude of $V_L$ and $V_{RL}$, it should be noticed first that phasor $V_R$ should be in phase with the reference current, while phasor $V_A$ has two phasor components: $V_{RL}$ which is in phase with the current and $V_L$ which leads the current by $90^o$. Next, since the phase angle of $V_A$, let’s call it $\theta$, must be between $0^o$ and $90^o$, a compass with a radius of $V_A$ and with its center point at the head of $V_R$ was used to draw an arc in the first quadrant. Similarly, a compass with radius $V_s$ and center point at the common origin was used to draw another arc in the first quadrant. Plotting $V_A$ from the head of $V_R$ to the intersection of the arcs and $V_s$ from the common origin to the intersection of the arcs yields the voltage phasor diagram that satisfies KVL for the circuit $V_s = V_R + V_A$. Careful measurement of $V_L$ and $V_{RL}$ from the phasor diagram combined with a knowledge of frequency of the source, a measurement of $I$ yields the values of $L$, $R_L$, and $Q_S$ of the inductor.

$$Q_s = \tan \theta = \frac{|V_L|}{|V_{RL}|} = \frac{|IX_L|}{|IR_L|} = \frac{X_L}{R_L}$$

The graphical technique of phasor diagram analysis may also be applied to R-L-C circuit operating in steady-state at a constant frequency. Graphical analysis of the series RL circuit of Figure 1(b) closely parallels the analysis of series RLC circuit of Figure 2(b). To simplify the analysis, assume that the capacitor is an ideal element. Hence, the voltage phasor $V_C$ may be plotted at –90 degrees relative to the circuit current $I$.

Graphical determination of circuit components such as $L$ and $R_L$ of Figure 1(a) by the phasor diagram method yields best results if the Q of the inductor is such that $\theta$ is in the range of approximately 30 to 60 degrees, or equivalently $Q_S$ is about 0.5 to 2. The Q of the inductor available for your use is, by itself, considerably greater than 2. This results in the angle associated with $V_A$ ($\theta$ in Figure 2) being so near to $90^o$ that avoiding significant errors in the graphical analysis is very difficult. Adding a 20 KΩ resistor in series with the inductor and treating the added resistor as part of the inductor’s resistance (making the combination a “lossy inductor” of low Q) results in a more practical laboratory situation.

Figure 2. (a) A series RLC ac circuit (b) Phasor Diagram for series RLC ac circuit
The Impedance Bridge, Models, and Quality Factor $Q$

Figure 3 shows the Series and Parallel Inductor models. The series $Q$ of a practical inductor or capacitor is equal to its parallel $Q$:

$$Q_s = Q_p = Q$$

where $Q_s = X_s/R_s$ and $Q_p = B_p/G_p$ for practical inductive or capacitive reactances. Also, continuing to work with the models of Figure 3 yields

$$R_p = R_s \left(1 + Q^2\right)$$

$$X_p = X_s \left(1 + \frac{1}{Q^2}\right)$$

If the nature of $X$ is inductive, then

$$L_p = L_s \left(1 + \frac{1}{Q^2}\right)$$

If the nature of $X$ is capacitive, then

$$C_p = C_s \left(1 + \frac{1}{Q^2}\right)$$

Figure 3. (a) Series inductor model (b) Parallel inductor model

PRELAB

Figure 4. Series RLC ac circuit
1. Draw the phasor diagram for the circuit shown in Figure 4 and determine the values of C, L, and $R_L$ for an operating frequency of 1000 Hz.
2. From your results of the previous part, determine the inductor’s parallel circuit model.

**PROCEDURE**

**MAKE EACH PHASOR DIAGRAM THE SIZE OF AN ENTIRE PAGE OF ENGINEERING PAPER.**

![Figure 5. Series RLC ac circuit](image)

Accurate results may be obtained graphically **ONLY** if great care is exercised in making the various voltage and current measurements AND in plotting the results.

**Part 1: Series RL Circuit**

1. Assemble the circuit of Figure 5 without the capacitor (R-L circuit only).
2. Set the function generator for **HIGH IMPEDANCE** and set $V_s$ to be sinusoidal at 1000 Hz.
3. Using multimeter across the 25 k$\Omega$ resistor, set the rms of $V_s$ to yield a convenient value of $V_R$ (such as 2.0 Vrms).
4. At your chosen value of $V_R$, measure and record the values of $V_s$ and $V_A$.
5. Construct an accurate full-page phasor diagram (similar to Figure 1(b)):
   - Draw current $I = V_R/R$ as the reference phasor (with $\theta^\circ$ phase on the horizontal axis)
   - Draw $V_R$ (in phase with $I$)
   - Use compass set to the measured value of $V_A$. Use the scale given on reference axis. Place the pin of compass at the tip of $V_R$ and mark an arc in the first quadrant.
   - Repeat the previous step for $V_s$ from the origin.
   - Draw point of intersection
   - Draw vector $V_A$ from tip of $V_R$ to point of intersection
   - Drop vertical line from point of intersection to intersect the horizontal line drawn from tip of $V_R$ (or tip of $V_C$). This is $V_L$.
   - By KVL $\vec{V}_A = \vec{V}_{RL} + \vec{V}_L$, therefore the horizontal component of $V_A$ is $V_{RL}$ and note that $V_{RL}$ should also be in phase with the reference current $I$.
6. From the diagram, measure $\phi$, $\theta$, $V_L$, and $V_{RL}$. Using these values, compute $R_L$, $X_L$, inductor’s Q and inductance, and the power factor of the circuit as seen by the function generator.
Part 2: Series RLC Circuit

1. Install the capacitor. With $V_S$ set to obtain the same value of $V_R$ as in part 1, measure $V_S$, $V_R$, $V_A$ and $V_C$.

2. Using the same current and voltage scales as you used for the phasor diagram of part 1, construct a full-page phasor diagram of the R-L-C circuit (similar to Figure 2(b)).

3. From the phasor diagram, measure and calculate, $\phi$, $\theta$, $V_L$ and $V_{RL}$. Using these measured values, compute $R_L$, $X_L$, inductor’s Q and inductance, and the power factor of the circuit as seen by the function generator.

4. Using the Impedance Bridge (select its internal 1 kHz oscillator), measure and record the inductance and Q of the equivalent low-Q inductor (the series combination of the 8-H inductor and 20-Ω resistor).

5. From these bridge measurements, calculate the series equivalent resistance of the 8-H inductor (the low-Q equivalent inductor’s resistance minus the 20 kΩ added resistance).

6. Compare and calculate the percent difference of the values of $R_L$ and $L$ calculated from the two phasor diagrams (part 1 & 2) to the bridge-based values obtained previously.